Gravitation and Gravity

Unit: 3.1

Every object in the universe attracts every other object with a force which is called the force of gravitation.

Gravitation is one of the four classes of interactions found in nature. They are

- the gravitational force
- the electromagnetic force
- the strong nuclear force (also called the hadronic force)
- the weak nuclear forces.

It is the gravity that holds the universe together.

Newton's Law of Gravitation:

The gravitational force acting between two point objects is proportional to the product of their masses and inversely proportional to the square of the distance between them.

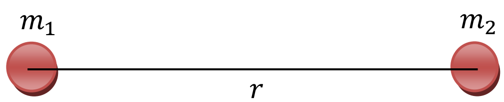

Fig. 3.1.1

Let, m_1 and m_2 be the masses of two objects separated by a distance r [Fig. 3.1.1]. The force (F) of attraction between these objects is

$$F \propto m_1 m_2$$

$$\propto \frac{1}{r^2}$$

$$\boxed{F = \frac{G m_1 m_2}{r^2}}$$

where G is universal gravitational constant.

The value of G is 6.67 X 10^{-11} Nm ^{2} kg $^{-2}$ and is same throughout the universe. The value of G is independent of the nature and size of the objects as well as the nature of the medium between them.

Acceleration due to gravity:

The uniform acceleration produced in a freely falling object due to the gravitational pull of the earth is known as *acceleration due to gravity*.

It is denoted by g and its SI unit is m/s^2. It is a vector quantity and its direction is towards the centre of the earth.

The value of g is independent of the mass of the object which is falling freely under gravity. The value of g changes slightly from place to place. The value of g is taken to be 9.8 m/s^2 for all practical purposes.
The value of acceleration due to gravity on the moon is about one $\frac{1}{6}$ th of that on the earth and on the sun is about 27 times of that on the earth.

The gravitational force experienced by a body of mass m due to the earth of mass M is

$$F = \frac{GMm}{r^2}$$

where r is the distance between the centre of the body and the earth.

Also, from Newton's second law of motion, we have

$$F = mg$$

Equating the above two forces,

$$\frac{GMm}{r^2} = mg$$

$$=> g = \frac{GM}{r^2}$$

This equation shows that g is independent of the mass of the body. But, it varies with the distance from the centre of the earth. If the earth is assumed to be a sphere of radius R, the value of g on the surface of the earth is given by

$$\boxed{g = \frac{GM}{R^2}}$$

Q.1 Find the mass of the earth, given that the radius of the earth is $6.4 x 10^6$ m and G=$6.67 x 10^{11}$ in SI units.

Variation of acceleration due to gravity with altitude (height):

Let P be a point on the surface of the earth and Q be a point at an altitude h [Fig. 3.1.2]. Let the mass of the earth be M and radius of the earth be R. We consider the earth to be a spherical shaped body.

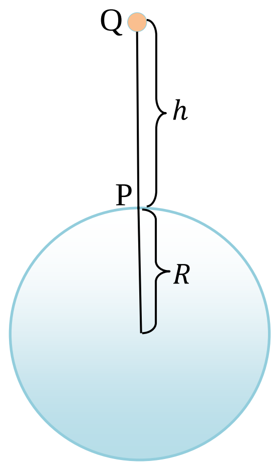

Fig. 3.1.2

The acceleration due to gravity at P on the surface of the earth is

$$g = \frac{GM}{R^2} \qquad (3.1.1)$$

Let the body be placed at Q at a height h from the surface of the earth. The acceleration due to gravity at Q is

$$g_h = \frac{GM}{(R+h)^2} \qquad (3.12)$$

Dividing equation (3.1.2) by (3.1.1), we get

$$\frac{g_h}{g} = \frac{R^2}{(R+h)^2}$$

By simplifying and expanding using binomial theorem, we get

$$g_h = g\left(1 - \frac{2h}{R}\right)$$

The value of acceleration due to gravity decreases with increase in height above the surface of the earth.

Variation of acceleration due to gravity with depth:

Let us consider the earth to be sphere of mass *M* and radius *R*. A body of mass *m* is placed initially on the surface and finally taken *x* distance deep into the earth [Fig. 3.1.3].

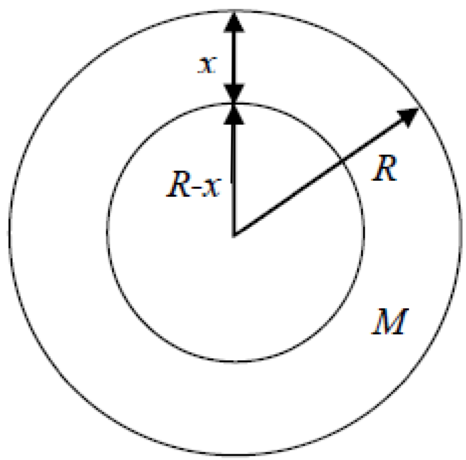

Fig. 3.1.3

We know acceleration due to gravity on the surface of earth is

$$g = \frac{GM}{R^2}$$

$$=> g = \frac{G}{R^2}\frac{4}{3}\pi R^3\rho$$, where ρ is the density of earth.

$$=> g = G\frac{4}{3}\pi R\rho \qquad (3.1.3)$$

Acceleration due to gravity at *x* distance deep is only due to mass M'.

$$g' = \frac{GM'}{R'^2} = \frac{G}{(R-x)^2}\frac{4}{3}\pi(R-x)^3\rho$$

$$=> g' = G\frac{4}{3}\pi(R-x)\rho \qquad (3.1.4)$$

Dividing equation (3.1.4) by (3.1.3), we get

$$\frac{g'}{g} = \frac{R-x}{R}$$

$$=> g' = g\left(\frac{R-x}{R}\right)$$

This expression shows that acceleration due to gravity decreases as we go deep into the earth. At the centre of earth $x = R$, so $g = 0$. Hence, the acceleration due to gravity at the centre of earth is zero.

Centre of gravity :

The centre of gravity of a body is defined as that point at which the body's entire weight can be regarded as being concentrated. A body can be suspended in any orientation from its centre of gravity without tending to rotate.

Centre of mass:

The centre of mass of a body is defined as that point which as though all of the mass were concentrated there and all external forces were applied there.

Escape velocity (v_e):

When we throw a body vertically upwards, it will return to the earth's surface after attaining certain height because gravitational force of attraction. When the velocity of projection is increased, the height attained by the body becomes greater. If the body is to be projected with a particular greater velocity, the body escapes from the gravitational pull so that it never returns to the earth. This velocity of projection is called escape velocity. It is different for different planets.

The minimum velocity required to project a body in upward direction from the surface of the earth so that it escapes from the earth's gravitational pull is called escape velocity. It is different for different planets.

The escape velocity of a body from the surface of the earth of mass *M* and radius *R* is given by

$$\boxed{v_e = \sqrt{\frac{2GM}{R}}}$$

This can also be written as

$$\boxed{v_e = \sqrt{2gR}} \qquad [\text{since, } g = \frac{GM}{R^2}]$$

It is be noted that the escape velocity of a body does not depend on its mass. The escape velocity from the surface of the earth is about 11.2 km/s.

Orbital velocity (v_o):

In order to put a satellite into the orbit around the earth, the satellite must be projected vertically upward to particular height and then it must be turned in a direction perpendicular to the line from the centre of the earth so that it moves in an orbit around the earth.

The velocity of a satellite along its orbit around the earth is called orbital velocity.

Let, *m* be the mass of a satellite revolving around the earth of mass *M* at a height *h* from the surface of the earth.

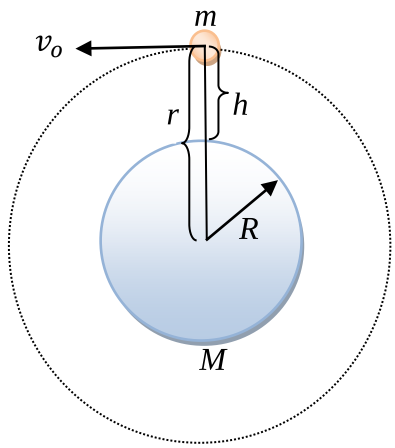

Fig. 3.1.4

The gravitational force of attraction between the earth and the satellite separated by a distance *r* is

$$= \frac{GMm}{r^2}$$

This provides the necessary centripetal force to the satellite to move in an orbit. If v_0 is the orbital velocity of the satellite, then we can write

$$\frac{mv_0^2}{r} = \frac{GMm}{r^2}$$

$$=> v_0^2 = \frac{GM}{r}$$

$$=> v_0 = \sqrt{\frac{GM}{R+h}}$$

where R is the radius of the earth.

Since $g = \frac{GM}{R^2}$, the expression for orbital velocity then becomes

$$\boxed{v_0 = \sqrt{\frac{gR^2}{R+h}}}$$

For a satellite launched in near earth orbit, $R + h \approx R$. In this case, the orbital velocity is

$$\boxed{v_0 = \sqrt{gR}}$$

Relation between escape velocity (v_e) and orbital velocity (v_0):

$$\frac{v_e}{v_0} = \frac{\sqrt{2gR}}{\sqrt{gR}} = \sqrt{2}$$

$$=> \boxed{v_e = \sqrt{2}v_0}$$

Artificial satellites:

Artificial satellites are human-built objects orbiting the earth and other planets in the solar system. This is different from the natural satellites (e.g. moon) that orbit planets.

Geo-stationary satellite:

A geo-stationary satellite or synchronous satellite is one which appear to remain in fixed position at a height of 36,000 km above the equator.

Television programmes or events occurring in other countries are often transmitted live with the help of these satellites.

Polar satellites:

The polar satellites revolve around the earth in a north-south orbit passing over the poles. The polar satellites positioned nearly 500 to 800 km above the earth travels from pole to pole in around 100 minutes. The polar orbit remains fixed in space as the earth rotates inside the orbit. As a result, excellent coverage of the earth is possible with the satellite in polar orbit.

The polar satellites are used for mapping and surveying.

Uses of artificial satellites:

The artificial satellites are launched for many purposes by different countries. The important uses of artificial satellite are:

- Collection of scientific data
- Weather monitoring
- Military spying
- Remote sensing
- Communication purpose – the satellite receives microwaves and TV signals from the earth and amplifies them and transmits them back to various stations on the earth